APPLICATION OF WEIBULL DISTRIBUTION ON DESIGN WAVE PARAMETER PREDICTIONS

Dissertation

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CERTIFICATE

*This is to certify that this dissertation titled "**Application of Weibull distribution on design wave parameter predictions** " is an authentic record of the work carried by Mr. Praveen V.K. In The Department of Physical Oceanography, under my guidance, in partial fulfillment of the requirements for the degree of Master of Science in Oceanography and that no part of it has been presented previously for the award of any other degree in any university.*

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DECLARATION

*I hereby declare that this dissertation titled "**Application of Weibull distribution on design wave parameter predictions**" is an authentic record of the work carried out by me under the guidance of Dr.G.Muraledharan, in the Department of Physical Oceanography and that no part of it has been presented for any degree, diploma or other similar titles in any university.*

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Praveen.V.K

To my parents…

CONTENTS

CHAPTER –1

INTRODUCTION

What are waves? How they are forming in oceans? How it changes its shape? These are the usual questions of mankind for centuries. Shortly we can say that, waves are the surface undulations of the sea, which varies in its behaviour from place to place and time to time. Its detailed study can yield many design parameters for the construction of marine structures. So that we can design structures, which withstand the forces and overtopping induced by these waves. The profile of wind-generated waves observed in the ocean changes randomly with time; it is repeatable in time and space. In reality, both wave height (peak-to-trough excursions) and wave period vary randomly from one cycle to another. It is often observed that waves break when the wave steepness exceeds a certain limit.

There is a growing need for surface wave information on the continental shelf and beach to estimate sea state, and to provide input for models of currents, sediment transport, radar backscatter and aerosol generation etc. While surface wave spectra in the open ocean vary slowly over distances of 100-1000km, wave properties on the continental shelf and beach are highly variable (typical length scales of 0.1-10 km) owing to a variety of topographic effects (e.g., shoaling, refraction, scattering) and strongly enhanced nonlinear interactions and dissipation. The long-term goal of this research is to develop a better understanding of the physical processes that affect the generation, propagation and dissipation of surface waves in shallow coastal waters, and improve the accuracy of models that predict the transformation of wave properties across the shelf and beach.

An important part of selecting the data with which to perform a prediction of long-term wave conditions is the definition of a specific population of interest (Borgman, 1977). In most cases the wave conditions to be predicted are associated with the more severe climatological events that occur in the study area. The climatological events may be from one or more storm populations such as hurricanes, tropical storms and winter storms. Wave conditions resulting from different populations must be analysed separately to avoid the effects of mixing on the results.

An extreme wave height poses severe consequences on coastal structures. How frequently the event of extreme wave heights may be expected to occur is of great importance. Design of civil engineering structures and insurance risk calculations, for instance rely on knowledge of the frequency of these extreme events. Estimation of these frequencies is however, difficult because extreme events are by definition rare and data records are often short. In other words, the uncertainties related to the distribution analysis of wave heights are high.

1.1 Probabilistic behaviour of ocean waves

When wave propagates, it can be considered as a random process (R.Silvester, 1974). It is found that when two wave records taken reasonably close together may in detail look extremely unlike. However their statistical properties would indicate the similarities of their source. It is from the perspective of statistics that some order emerges from the seeming chaos of the sea surface. So that it adds some accuracy to data, which may be unwarranted. In statistical approach, wave parameters such as wave height, wave period etc. is considered as random variables, which possess some statistical characteristics.

Statistically meaningful understanding of random phenomena as waves requires a definite system of analyses through appropriate statistical techniques (Muraleedharan, 1991). To analysis such a random process the important statistical functions used are,

1. Probability density function

2. Mean values

3. Auto correlation function

4. Spectral density function

In this work wave distribution pattern is studied by probability density functions and derived therefrom-parametric relations of certain important wave statistics. The mathematical models like Weibull, Rayleigh, Exponential and Gumbel are used to understand the wave statistics by using the visual data covering 10 years from 1960-1969 (Deep water wave data) and then by recorded data for Alleppey during monsoon season from 1981 to 1984 (Shallow water wave data).

The spatial and temporal statistics of breaking waves (ie.H_{max}) such as duration, velocity, spacing, and breaking probability were determined under various wind and wave conditions. Statistical models are developed to assess and where appropriate, correct for any bias resulting from limitations of the measurement approach. Empirical relations of these statistics with wind speed are obtained. Comparison of the observed distributions with simultaneously measured directional wave spectra suggests that wave breaking occurs at multiple scales and that the mean scale of breaking is substantially smaller than the scale associated with the dominant wind wave component. Preliminary analysis indicates that the dependence of breaking probability on the fourth moment of the wave spectrum is consistent with a linear statistical model.

Related to the statistical uncertainty is the problem of the limited accuracy of physical mathematical models. In many cases the mathematical model of a physical process has to be calibrated by means of small-scale experiments or prototype observations. Due to the limited number of experiments or observations and to imperfections of the model, differences will exist between the predicted and observed outcomes. These differences give rises to the model uncertainty. Model uncertainty is related to the statistical uncertainty in the sense that more data tend to reduce the parameter uncertainty. In case of an imperfect model some uncertainty will persist unless the model itself is improved.

1.2 Stochastic prediction of wave characteristics

Stochastic prediction of various properties of random waves (height, period, etc.) is done by the development of probability function applicable for each of these properties (Michel K. Ochi, 1998). The input parameters for this probability function are evaluated by using programs.

There are two different approaches to acquire input information; one in the time domain, the other through the frequency domain. In the statistical properties of wave heights, the necessary input to the probability function can be evaluated by reading all individual wave heights from measured data, and estimating the parameters of the probability function based on statistical inference theory. This approach for evaluating the parameter(s) of the probability function is called the random observation method in time domain. The reliability of estimated results by this method is not high unless the number of observations is sufficiently large. For a non-narrow banded spectrum, the most commonly considered probability distribution for wave amplitude is developed assuming that the wave height is twice the amplitude. Especially the significant wave height can be used for representing the severity of sea conditions. It is defined as the average of the one-third highest observed or measured wave heights.

1. 3 Long-term wave height distribution

Long-term wave statistics plays an important role for the design of marine systems, since the accumulation of response of a marine system in each short-term sea state over its lifetime provides information vital for evaluating fatigue loads on the system.

The long-term statistics of individual wave height is an accumulation of the statistics for all short-term sea conditions, taking in to account the frequency of occurrence of each short-term sea state.

1.4 Earlier studies

Most of the wave studies in India are carried out in the West Coast. Considerable amount of ship-observed data reported by IMD have been analysed (Srivastava and George, 1976; Thiruvengadanathan, 1984). Two important contributions that emerged during this period were, the atlases prepared by Naval Physical and Oceanographic Laboratory (1978) and National Institute of Oceanography (1982).

Dattatri, Raman and Jothi Shankar (1979) studied the height and period distribution for waves off Mangalore harbour.

Dattatri and Renukaradhya (1971) made hindcasting of waves along the west coast of India using SMB and PNJ methods. The predicted wave heights were found to be consistently higher than the observed.

Shahul Hameed (1988) studied the wave climatology and littoral processes at Alleppey. He found out many parameters like H_{max}, H_s, wave directions etc. of that location.

Muraleedharan, Nair and Kurup (1988) have observed the averaged visual wave statistics for the southwest coast of India. From the studies they found that the best fit for wave heights are obtained for the Weibull probability density function.

Baba and Harish (1985) studied spectral and statistical properties of shallow water waves off southwest coast of India. Swami et.al (1976) analysed wave characteristics off Goa.

The main source of long term wave data are the atlases prepared using the wave data reported by ships (visually estimated) and provided by the Naval Physical Oceanographic Laboratory (1978) and National Institute of Oceanography (1982).

Thomas (1988) described the various wave parameters of shallow water waves off Valiathura. The waves were always above 0.5 m in height and the maximum wave height (H_{max}) observed was 6.0 m during 1980-1984.

Muraleedharan (1991) analysed 348 sets of data from NPOL (1978) and NIO (1982) wave atlas and concluded that the Weibull model had logical as well as experimental support for explaining the long-term wave height distribution patterns. Muraleedharan et al., (1999) redefined the significant wave height as the average of the highest $1/3^{rd}$ of a constant number of consecutive individual zero up crossing wave heights in a wave record and suggested the Weibull model for simulating the distribution of the redefined significant wave height by the method of characteristic functions and for predicting various wave statistics.

Unnikrishnan Nair et al., (2002) redefined the significant wave period and derived prediction formula for various redefined wave period parameters from the modified Erlang model suggested for redefined significant wave period distribution by the method of characteristic functions.

Battjes, (1970), investigated the probability distributions of the significant wave heights on a long-term time scale (e.g. one year). He noticed that symmetric distributions, such as the normal distribution, were not suitable to describe the long-term distribution for the wave heights. Skewed distributions, such as the Gumbel and Weibull distribution, fitted much better.

Longuet-Higgins, (1952), investigated the probability distributions of the individual wave heights on a short-term time scale. With theoretical arguments, he derived that the short-term distributions is given by a Rayleigh distribution, given a few easily satisfied, boundary conditions (such as stationary conditions) and he used the free parameter in the Rayleigh distribution as conventional significant wave height.

Roelvink (1993) studied the spatial distribution of the mean wave energy using parametric models and concluded that the Weibull distribution provided the most accurate and the Rayleigh yielded the least accurate results. Vijay Panchang et al., (1997) analyzed the extreme wave statistics for Gumbel and concluded that if distribution models such as Weibull or Lognormal, which are found to be more effective in describing the sea state, are used for extreme value analysis then the result will be somewhat different.

1.5 Present study

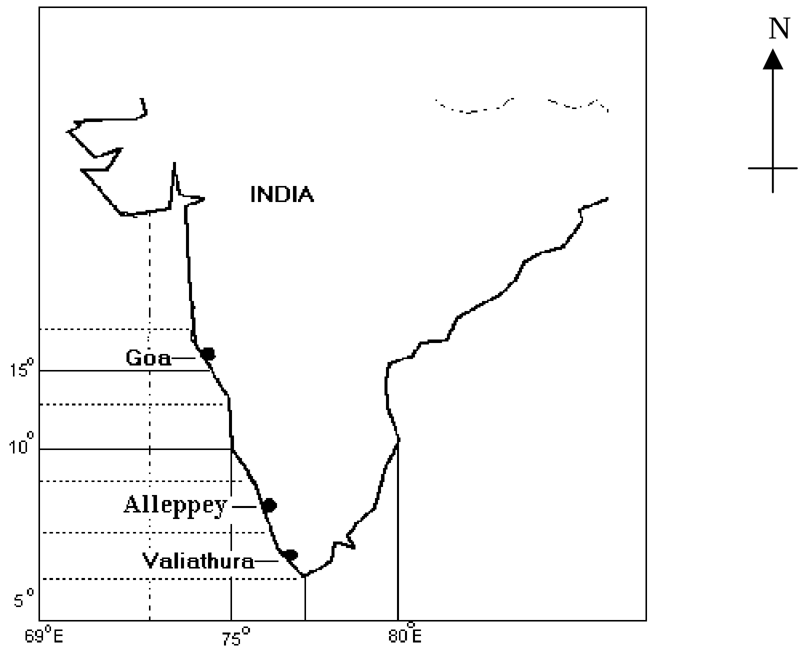
Present study includes the probability model behaviour on ocean waves by using different models like Weibull, Rayleigh, Exponential and Gumbel, which are the available model for long-term distribution of wave heights. The NPOL data at Goa is used for proving the theoretical supremacy of Weibull model over other competing models.

The shallow water maximum wave height distribution recorded off Alleppey is simulated by a modified Weibull model used for predicting maximum wave parameters like, Extreme Wave Height, Most Frequent Maximum Wave Height, Mean Maximum Wave Height and probability of realizing an Extreme Wave Height in a time less than the designated return period. The analysis is done for six months (rough monsoon seasons, May to October) from the clubbed data of 1981-1984.

Since the data is only for four years prediction is also done for the next four years since shallow water conditions are less consistent and hence prediction for more years will not be reliable.

The digitized zero up-crossing wave height distributions off Valiathura are simulated by Weibull model and the predicted conventional significant wave height (average height of the highest one-third waves, Sverdrup and Munk, 1947) was compared with the computed and the comparability was verified by RMS, Relative RMS and mean RMS errors for January 1981 data.

AREA OF STUDY



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CHAPTER– 2

MATERIALS AND METHODS

2.1 Source of present data

The visually estimated wave heights and periods (NPOL, 1978), instrumentally recorded wave data (CESS, 1984) and data extracted from recorded wave chart by using a digitising software package, ACECAD (Roshni.M.G., Jayesh.C.K, Santhoshkumar.T.A, 2000) are considered in this work.

The visually estimated wave data (Direction, Period and height of the waves) have been compiled from the weather charts for the area under reference for 10 years from 1960-1969. The data off Goa (Zone No: 9, Lat. 13^0N-17^0N, Long. 73^0E-77^0E) for 12months is used for fitting Weibull, Rayleigh, Exponential and Gumbel distribution models, which are the available competing models for wave height distribution.

Instrumentally recorded wave data for months from May to October during the year 1981 to 1984 are used in the present study. A pressure type wave and tide telemetering system and a wave rider system are used for the recording. The sensor is installed at a depth of 5.5m. A timer is tuned to record the waves for 30 minutes during every 3 hours. The record is then subjected to Tuckers analysis and the digitised maximum wave height distribution during this period is considered for this study since we can expect large waves in monsoon seasons. Since Alleppey coast (Lat.8^0N-12^0N, Long.75^0E-77^0E) is shallow, a depth factor $\frac{\overline{H}_{Max}}{depth}$ is also included in the modified Weibull model derived for maximum wave height distribution for accommodating shallow water attenuation effects. Various parametric relations have been derived from this model for predicting maximum wave height parameters such as mean maximum wave heights, most frequent maximum wave height, extreme wave height, return period of an extreme wave height and probability of releasing an extreme wave height in a time less than the designated return period.

Recorded wave chart data at 0090 hrs, 1200 hrs and 1500 hrs off Valiathura derived from a digitiser called ACECAD is used for compiling individual zero up-crossing wave heights for the month of January 1981. This method uses the zero-up crossing technique. The accuracy of such digitised data is very high since in

ACECAD 1cm is divided into 200 minor divisions. Zero-up crossing wave height and period are obtained from the digitised data. The distributions of zero up-crossing wave heights are simulated by the depth factor included Weibull density function.

2.2 Goodness-of-fit test

χ^2- test at 0.05 level of significance is applied to the wave data for goodness of fit test. This test establishes the confidence with which a sample of observed frequencies of occurrence of particular events could be assumed to belong to a theoretical distribution. The degrees of freedom of the data is given by,

$$DOF = N-K- (K-1)$$

Where N = Total number of classes

K = Total number of parameters used in the probability distribution.

We can find out the χ^2 value by using the equation

$$\sum \frac{(of_i - ef_i)^2}{ef_i}$$

Where of_i = Observed frequency

ef_i = Effective or Computed frequency

If the computed value is less than the table value for the given degrees of freedom and level of significance the fittness of the data with the model is accepted.

2.3 Modelling of waves

a) Weibull distribution

The Weibull distribution was discovered as early as the latter half of the twenties in a discussion relating to the asymmetric distribution of the extreme values in sample (Muraleedharan, G., 1991). This distribution is one of the most widely used lifetime distributions in reliability engineering. It is a versatile distribution that can take on the characteristics of other types of distributions, based on the value of the shape parameter, “b”. This model is used for simulating the long-term distributions of the wave heights. A random variable X possess a Weibull distribution if it has a probability density function of the form

$$f(h) = \left(\frac{b}{a}\right)\left(\frac{h}{a}\right)^{b-1} \lambda^{-\left(\frac{h}{a}\right)^{b}}$$

a-scale parameter, b-shape parameter, h-variable.

a,b,h>0.

The cumulative distribution function is,

$$d.f = 1 - \lambda^{-\left(\frac{h}{a}\right)^{b}}$$

b) Rayleigh distribution

Only Rayleigh distribution has been derived mathematically on the basis of physical processes governing wave systems from various theoretical models. Rayleigh distribution is based on the following assumptions (Muraleedharan, G., 1991).

1.The distribution of instantaneous ocean surface elevation is assumed as a Gaussian normal.

2.Wave energy is confined to a narrow range of frequencies.

3. Waves are the result of superposition of many sinusoidal components in random phase.

These conditions are not always satisfied. The main reasons can be attributed to natural conditions that do not meet the assumptions (1), (2) and (3) specified above. Contributions from the different parts of the generating area are to be superposable means that the mechanical system we are dealing with is linear, which is true generally for low waves in deep water and not for waves tending to attain maximum height. Hence we need a model that can meet the twin objectives of accommodating the Rayleigh distribution whenever the basic assumptions that justify it are satisfied, and fitting data situations under more general conditions. One way of achieving this is to make the Rayleigh distribution model more flexible by adding one more parameter to its density function given by,

$$f(h) = \left(\frac{2}{a}\right)\left(\frac{h}{a}\right) \lambda^{-\left(\frac{h}{a}\right)^{2}} \times dh$$

The universal constant ‘2’ in the factor exp-$(h/a)^2$ means that the curve can provide only the same shape irrespective of the values of ‘a’ which changes only the scale. This motivates the use of an arbitrary positive quantity ‘b’ in the place of ‘2’ in the exponent to arrive at Weibull density function.

Thus the modified curve becomes Rayleigh whenever b=2 and our twin objectives stated above are satisfied. Apart from accommodating curves of different shape in the modification suggested above, another possible explanation for the Weibull curve can be offered in terms of the intensity function described below (Muraleedharan, 1991).

Let 'H' be the continuous random variable representing the wave height in a specified region taking values in the interval (0,W) where 'W' is the maximum height to which wave can rise. For a given height 'h', the probability that 'H' exceeds 'h' is

$$P(H > h) = 1 - F(h)$$

Where F(h) is the distribution function of H. The probability that a wave of height 'h' decays before height h+dh is attained (the decay takes place within an infinitesimal height dh) is

$$P\left(\frac{h < H < h+dh}{H > h}\right) = P\left(\frac{h < H < h+dh}{P(H > h)}\right) = \frac{f(h)dh}{1-F(h)}$$

where f(h) is the density function of 'h'. If the left side of the expression is denoted by k(h)dh, k(h) represents the intensity at which a wave of height 'h' begins to decay. From the knowledge of the form of the function, one can determine the distribution of 'H' as (Gumbel, 1954),

$$f(h) = k(h).\lambda^{-\int_0^h k(t).dt}$$

For the Longuet-Higgins (1952) model of wave heights

$$K(h) = \frac{2h}{a^2}$$

So that the intensity of decay is directly proportional to the height of the wave. On the other hand for the Weibull model

$$K(h) = \left(\frac{b}{a^b}\right)h^{b-1} \qquad ; a, b > 0$$

The rate at which the intensity changes is dk(h)/dh. So that for the Rayleigh distribution it is $2a^{-2}$,which is a constant. In the Weibull case it becomesb (b-1)a^{-b}.h^{b-2}, which varies with h.

The main point here is that the proportion of waves that decays when they passes height 'h', increases as the value of 'h' increases in both the models. While for Rayleigh model this increase in proportion for unit change in height remains the same, for the Weibull case the proportion decaying increases at a faster rate. (Here we assume that the parameter b>1, which is justified empirically). In other words the proportion decaying at high values of 'h' is much greater than the proportion of waves decaying at smaller heights. This appears to be more realistic than a uniform increase in proportion according to the Rayleigh model, especially when the narrow band assumption does not hold.

In any case, it is important to note that the assumption of Weibull law does not contradict the Longuet-Higgins conditions, which is automatically satisfied in situations favourable to it, but only supplements it by extending the condition to more general situations.

Once the long-term distribution of wave heights is assumed to be Weibull, it is possible to have specific expressions for the different wave parameters that are used to predict the various design wave parameters.

In the above models "b" is the solution of the equation given below for g (b) = 0

$$g(b) = \frac{\sum_i x^b \ln x_i}{\sum_i x^b} - \frac{1}{b} - \frac{1}{n}\left(\sum \ln x_i\right)$$

Where 'n' is the total frequency. From the value of "b", the value of "a" can be found out by using the equation

$$a = \frac{\overline{h}}{\Gamma\left(\frac{b+1}{b}\right)}$$

Where $\overline{h}$ is the mean wave height

c) Exponential distribution

This model is used for examining the long-term distributions of the wave heights (Muraleedharan, G., 1991). The probability density function of this model is obtained by giving b = 1 in the Weibull distribution so that its density function will be,

$$f(h) = \left(\frac{1}{\overline{h}}\right)\lambda^{-\left(\frac{h}{\overline{h}}\right)}$$

The cumulative distribution function is,

$$d.f = 1 - \lambda^{-\left(\frac{h}{\overline{h}}\right)}$$

d) Gumbel distribution

This model is used for examining the long-term distributions of the wave heights. The probability density function of this model is given as (Muraleedharan, G., 1991)

$$f(h) = \left(\frac{1}{\theta}\right)\lambda^{-\left(\frac{h-u}{\theta}\right)} \times \lambda^{-\left(\lambda - \left(\frac{h-u}{\theta}\right)\right)}$$

The cumulative distribution function is given by,

$$d.f = 1 - \lambda^{-\lambda^{-\left(\frac{h-u}{\theta}\right)}}$$

Where θ and U are the parameters of the model. In order to estimate these parameters we use the method of moments to obtain estimators.

$$\theta = S\sqrt{\frac{6}{\pi}}$$

Where S is the sample standard deviation and

$$u = \overline{h} - r\theta$$

Where “r” is the Euler’s constant given by

$$r = Lt_{n\to\infty}\left(1 + \frac{1}{2} + \frac{1}{3} + \dots\dots\dots\dots\dots\frac{1}{n} - \ln(n)\right)$$

r ≈ 0.5772157

2.4 Estimation of wave parameters for the Weibull model

The parameters ‘a’ and ‘b’ are computed using suitable computer program (FORTRAN 77). In the present study, the wave height is tabulated for class intervals of 0.2m.

2.5 Application of Weibull model for ocean wave height data

The Weibull model can be used for simulating shallow water wave height distribution by appropriately including shallow water coefficients like, shoaling, frictional, refraction, etc. (Muraleedharan, 1998). For Alleppey the values of the above coefficients are, 0.90-1.40, 0.43-0.92 and 0.67-1.34 (Kurian, 1987).

For simulating the maximum wave height distribution in the coastal waters off Alleppey, a depth factor is included in the modified Weibull model for maximum wave heights, whose distribution function is,

$$d.f = \left[1 - \lambda^{-\left(\frac{h}{a} + \frac{\bar{h}}{d}\right)^{b}} \right]^{n}$$

where n is the sample size

Certain parametric relations derived from this model (Muraleedharan et al., 2001) for predicting the design wave height parameters are given below.

a) Extreme wave height ($H_{extreme}$)

The prediction formula for extreme wave height is derived from the modified Weibull model for maximum wave height, whose density function is,

$$f(h) = n\left[\left(1 - \lambda^{\left(\frac{h}{a} + \frac{\bar{h}}{d}\right)^{b}}\right)^{n-1} \times \left(\lambda^{-\left(\frac{h}{a} + \frac{\bar{h}}{d}\right)^{b}}\right)\right] \times \frac{b}{a} \times \left(\frac{h}{a} + \frac{\bar{h}}{d}\right)^{b-1}$$

$$H_{extreme} = a\left[\left[-\ln\left[1 - \left[1 - \frac{1}{R_p}\right]^{\frac{1}{N}}\right]\right]^{\frac{1}{b}} - \frac{\bar{h}}{d}\right] \text{.......... ....} A(1)$$

where $\frac{\bar{h}}{d}$ *is called depth factor*

'a' is the scale parameter, b the shape parameter, n- sample size, R_p-Return period(in years), N-number of observations of the largest wave(in days) , d is the water depth. For practical purpose, for class sizes of 0.2-0.4, 0.4-0.6(for May to September maximum wave height data) etc., n is taken to be '2' in the concept that there are two wave heights in each class. This concept is also empirically established. For class sizes of 0.1-0.2, 0.2-0.3 (for October maximum wave height data to achieve sufficient number of classes) etc., n is taken to be '1'.

b) Return period (R_p) and Probability of realising an Extreme wave Period in a time less than the designated return period.

The Return period of an extreme wave height is derived from the equation A(1) as,

$$R_p = \left[1 - \left(1 - \lambda^{-\left(\frac{h_{extreme}}{a} + \frac{\bar{h}}{d} \right)^b} \right)^N \right]^{-1}$$

Where $\bar{h}$ is the mean wave height

Probability of realising an extreme wave height in a time (m) less than the designated return period is given by the equation,

$$P = 1 - \left[1 - \left(1 - \lambda^{-\left(\frac{h}{a} + \frac{\bar{h}}{d} \right)^b} \right)^N \right]^M$$

Where $M = m/R_p$, $m<R_p$

Here this probability is converted to their percentage value.

c) Most frequent maximum wave hieght (H_{mfm})

The Most Frequent Maximum height is given as,

$$H_{mfm} = a\left[\left(1 - \frac{1}{mb} \right)^{\frac{1}{b}} - \frac{\bar{h}}{d} \right]$$

d) Mean of maximum wave height $\left(\overline{H}_{Max}\right)$

$$\overline{H}_{max} = \left(\frac{a}{b} \right) \left[\frac{m\Gamma\left(\frac{1}{b}\right)}{1!} - \frac{m(m-1)\Gamma\left(\frac{1}{b}\right)}{2! \times 2^{\frac{1}{b}}} + \dots\dots\dots\dots \; \dots\dots + \frac{(-1)^{r+1}\Gamma\left(\frac{1}{b}\right)}{m^{\frac{1}{b}}} \right]$$

m = Sample size = 2, except for October, m = 1

2.6 Application of Weibull model for conventional significant wave height predictions

The individual wave heights digitised by the zero up-crossing technique (Roshni.M.G., Jayesh.C.K, Santhoshkumar.T.A, 2000) off Valiathura coast for the month of January'1981 is simulated by the Weibull distribution model after accommodating a depth factor and hence the parametric relation derived (Muraleedharan et al., 1991) from the model for predicting conventional significant wave height is considered for estimating H_s.

a) Conventional significant wave height

The conventional significant wave height is predicted using the expression,

$$H_{s(1/3)} = a[(\ln 3)^{1/b} - DF\text{-}SC] + (a/b)\, 3\, I_{\ln 3}(1/b) \;\dots\dots\dots\dots (2.7a)$$

Where DF is the depth factor and SC is shoaling Coefficient

The term $I_{\ln 3}$ (1/b)(Incomplete Gamma function) can be calculated by using the equation,

$$I_{\ln 3}\left(\frac{1}{b}\right) = \Gamma\left(\frac{1}{b}\right) - I(u, p)$$

$\Gamma\left(\frac{1}{b}\right)$ represents the gama value for 1/b ,which can be taken from Gamma Table.

$$Where\ P = \frac{1}{b} - 1$$

$$u = \sqrt{b} - 1$$

The value $I_{u,p}$ can be found out from the incomplete Gamma function table for u and p values

b) RMS error, Relative RMS error and Mean RMS error

The RMS_{error}, relative RMS_{error} and mean RMS_{error} are computed to provide the accuracy of the predicted values. Computer programs are developed to compute the various above mentioned wave height parameters and RMS_{error} . Given by,

$$RMS_{error} = \sqrt{\frac{(H_{comp} - H_{observ})^2}{N-2}}$$

$$\text{Relative RMS}_{error} = \frac{\sqrt{\frac{1}{N}\sum(H_{comp} - H_{observ})^2}}{\frac{1}{N}\sum H_{observ}}$$

$$\text{Mean RMS}_{error} = \frac{\sum(H_{comp} - H_{observ})}{\sum H_{observ}}$$

where H_{comp} is the theoretically computed wave height

H_{observ} is the measured (observed) wave height

N is the total no. of wave heights.

CHAPTER– 3

RESULTS AND DISCUSSIONS

3.1 Performance of different models for ocean wave height simulation

Visually estimated long-term conventional significant wave height distributions (NPOL, 1978; 10 years data (1960-1969)) for the grid.no.9 including Goa region are simulated using the available models; Weibull, Gumbel, Rayleigh and exponential. The simulation capability of each model is discussed here. The observed distributions of wave heights are shown as histograms and the theoretical simulations as curves (Figs.1-12). The theoretical supremacy of Weibull model over other competing models is reaffirmed by χ^2-test at 0.05 level of significance (Tables. I-XII). Weibull fits in 100% cases after grouping. It could be seen that the shape parameter of Weibull is never equal to '2', revealing that the sea state deviates from the basic assumptions of Longuet-Higgins model since these are long-term wave height data. The observed and theoretical properties of the conventional significant wave height distribution in grid No.9 (deep water) over a year are discussed below.

January

The wave height ranges from 0.4m to 4.5m. The scale parameter 'a' takes the value 1.08 and the shape parameter 'b' that takes care of the changing wave condition has a value 1.54. Weibull model simulates the higher side of the wave height distribution more effectively, Rayleigh and Gumbel to some extent. The Weibull curve is exponential and it simulates the higher side of the maximum wave height distribution better. A single peak for smaller wave height range shows the presence of swell waves

February

The wave height ranges from 0.4m to 2m. The scale parameter 'a' takes the value 0.96 and the shape parameter 'b' has the value 1.97. Both Weibull and Rayleigh simulate the wave height distribution pattern as it is evidenced from the shape parameter value, and Gumbel to some extent. As in the case of January the Weibull curve is exponential. Here also the observed wave height distribution shows only a single peak, which is an evidence of swell dominated sea state.

March

The wave height ranges from 0.4m to 4m. The scale parameter ‘a’ and the shape parameter ‘b’ have the values 1.14 and 1.46 respectively. Gumbel model explains the wave height pattern more effectively than the Weibull and Rayleigh. The observed wave height distribution shows only a single peak. Here the Weibull curve is exponential. Since there is no any secondary peak this again reveals the swell prevailed sea condition.

April

The wave height ranges from 0.4m to 4.5m. The scale parameter ‘a’ and the shape parameter ‘b’ have the values 0.95 and 1.67 respectively. The wave height distribution takes the form of a Weibull curve, Rayleigh and Gumbel to some extent. The observed wave height distribution shows only a single peak, and also the presence of higher waves are comparatively low, which is an evidence of the predominance of swell waves.

May

The wave height ranges from 0.4m to 5m. The scale parameter ‘a’ and the shape parameter ‘b’ have the values 1.16 and 1.41 respectively. Weibull and Rayleigh models simulate the observed distribution wave height patterns whereas Gumbel to some extent. The observed wave height distribution is double peaked, i.e. it shows a primary and a secondary peak. Compared to other months the higher side of the wave height distribution is more prominent. Here also the Weibull curve is exponential. This distribution shows the combined effect of both sea and swell waves. This shows the onset of monsoon.

June

The wave height ranges from 0.4m to 5m. The scale parameter ‘a’ and the shape parameter ‘b’ have the values 1.76 and 1.69 respectively. The Weibull curve fits the observed distribution wave height pattern better than the other models. The distribution of observed wave height shows a more complex nature than the previous months. The distribution is multi peaked. Higher waves are more frequent as evidenced from the wave height distribution. Here the Weibull curve is positively skewed. This is an evidence of sea-dominated state, which reveals the active monsoon period.

July

The wave height ranges from 0.4m to 7m. The scale parameter ‘a’ and the shape parameter ‘b’ have the values 2.16 and 1.83 respectively. Weibull, Gumbel and Rayleigh models fit the observed wave height distributions more or less accurately, whereas the Gumbel distribution estimates the peak frequency more effectively. Here also the Weibull curve is positively skewed. The distribution of observed wave heights shows multi peakedness with a broad range, i.e., a condition of a fully arisen sea state.

August

The wave height ranges from 0.4m to 4m. The scale parameter ‘a’ and the shape parameter ‘b’ have the values 1.80 and 2.15 respectively. Weibull curve follows the observed wave height distributions satisfactorily, Gumbel weakly simulates the lower side of the observed wave height distribution pattern whereas Rayleigh to certain extent. The wave height distribution is again multipeaked with a broad range, i.e., a condition of a fully developed sea state. Here again the Weibull curve is positively skewed. There are two primary peaks of nearly same magnitude in the height range 0.00-1.75m. The distribution is also predominant for higher wave heights, an evidence of active monsoon season.

September

The wave height ranges from 0.4m to 4.5m. The scale parameter ‘a’ and the shape parameter ‘b’ have the values 1.25 and 1.47 respectively. The Weibull model satisfactorily explains the observed wave height distribution whereas Gumbel and Rayleigh to some extent. The observed wave height distribution shows a primary and secondary peak. The distribution is not prominent for higher waves. Here again the Weibull curve is exponential. This shows the presence of moderate sea waves. This reduction from a complex distribution to a moderate one shows the initial stages of the retrieval of monsoon.

October

The wave height ranges from 0.4m to 3m. The scale parameter ‘a’ and the shape parameter ‘b’ have the values 0.98 and 1.87 respectively. Rayleigh model simulates the higher side of the observed wave height distribution, Weibull and Gumbel to some extent. The distribution of observed wave height is single peaked. Here the Weibull curve shows exponential behaviour. This shows the presence of swell waves. The rough monsoon season with sea waves has retrieved.

November

The wave height ranges from 0.4m to 4m. The scale parameter ‘a’ and the shape parameter ‘b’ have the values 1.05 and 1.50 respectively. Gumbel curve takes the shape of the higher side of the observed wave height distribution pattern, Rayleigh and Weibull to certain extent. Here the Weibull curve is again exponential. The distribution of observed wave heights shows only a single peak, a condition of swell waves and the distribution is less prominent for higher waves.

December

The wave height ranges from 0.4m to 2.5m. The scale parameter ‘a’ and the shape parameter ‘b’ have the values 0.88 and 1.80 respectively. The observed wave height distribution follows the Weibull curve, Rayleigh and Gumbel comparatively simulate weakly. Here the Weibull curve is exponential. The observed wave height distribution shows again a single peak with a narrow range of wave heights, a swell dominated sea state.

Weibull model shows 100% fitness by χ^2-test at 0.05 level of significance. Even though Gumbel and Rayleigh give some fruitful results, these models fail to explain the higher side of the observed wave height distribution. Weibull holds good results even in monsoon season where surface waves are high.

Muraleedharan et al., (1999) suggested the Weibull model for redefined significant wave height distribution and predictions by the method of characteristic function. A modified Gamma distribution model is suggested by the method of characteristic function for redefined significant wave period distribution and parametric relations were derived for predicting various redefined significant wave period parameters from Erlang distribution model (Unnikrishnan Nair et al., 2003).

Giving logical and empirical explanation for the zero up-crossing periods to follow a Gamma law, a mathematical expression is derived to simulate the various mean wave periods and its simulation capability is validated by comparing with field data using relative RMS and mean RMS errors. Then the Erlang distribution model is mathematically and empirically suggested for the conventional significant wave periods. (Unnikrishnan Nair et al., 2003).

Since it is difficult to find out a model for conventional significant wave heights, it is to be concluded from the above findings that the Weibull distribution itself will be a model for the same. Empirical findings also support the same view point (Muraleedharan et al., 1991). Since the waves reported (NPOL, 1978) are conventional significant waves, this study also reveals that the Weibull model effectively simulates the conventional significant wave height distributions.

3.2 A modified Weibull model for shallow water maximum wave height distributions and predictions

The maximum wave height distributions off Alleppey during monsoon seasons (May-October) are simulated after including a depth factor $\left(\frac{\overline{H}}{\text{depth}}\right)$ in the modified Weibull density function to accommodate the shallow water wave transformation effects. Frictional, shoaling and refraction coefficients show little effect on maximum wave height distributions. The observed distributions of wave heights are shown as histograms and the theoretical simulation as curves (Figs.13-18). In the month of October, since distribution of maximum wave height range is low, the class interval is made small to 0.1m. There is 100% acceptance by χ^2 test at 0.05 level of significance.

Design wave parameters such as most frequent maximum wave height, mean maximum wave height, extreme wave height, return period of an extreme wave height and probability of realising an extreme wave height in a time less than the designated return period are also estimated by the parametric relations derived from the model. A maximum deviation of 0.56m and a minimum of 0.06m for most frequent maximum wave height is obtained (Tables A to C). The corresponding deviations for extreme wave heights and mean maximum wave heights are 0.63m, 0.32m and 0.01m, 0.00m. As this is a long-term prediction (4 years) the maximum deviations of the predicted design wave parameters from the computed are in the appreciable limit.

Since the data is only for four years and it is for shallow waters, which are subjected to seasonal topographic changes, the prediction is also carried out to the next four years.

The extreme wave height distribution takes a slightly curved path (exponential curve)(Fig.19a-24a) where as the probability of realising an extreme wave period in a time less than the designated return period takes a straight-line path (Fig.19b-24b). Muraleedharan et.al (1998) had shown that if the predictions are for sufficiently longer periods then both the curves are clearly exponential, i.e., if the return periods anticipated are larger and larger, the increase in extreme wave height is exponential. Also for a given return period, the increase in probability percentage of realising an extreme wave period for an increase in time less than the designated return period is also exponential. For the month of July; return period, extreme wave heights, most probable maximum wave heights, etc. are predicted only for three years since the given data is available only for three years.

May

The maximum wave height ranges from 0.7m to 2.3m. The scale parameter ‘a’ takes the value 1.303 and the shape parameter ‘b’ that takes care of the changing wave condition has a value 3.198. The maximum wave height distribution is single peaked and have a broad range. The modified Weibull curve is positively skewed and it simulates the higher side of the maximum wave height distribution better. A single peaked maximum wave height distribution is an evidence of the predominance of the swell waves even though there is the presence of sea waves as an indication of the onset of monsoon.

June

The maximum wave height ranges from 0.5m to 2.7 m. The scale parameter ‘a’ and the shape parameter ‘b’ have the values 1.752 and 2.980 respectively. The distribution of maximum wave height is multipeaked describing the sea wave prevailed state. The distribution of higher waves is also prominent. This is due to the effect of monsoon and there by the generation of sea waves. Here the modified Weibull curve is normal.

July

The maximum wave height ranges from 0.3m to 2.7m. The scale parameter 'a' and the shape parameter 'b' take the values 1.518 and 2.931 respectively. The distribution of maximum wave height is also multipeaked showing the active presence of sea waves. The distribution is broad ranged. The modified Weibull curve is normal and it simulates the higher maximum wave height distribution better.

August

The wave height ranges from 0.3m to 2.7m. The scale parameter 'a' and the shape parameter 'b' have the values 1.410 and 2.672m respectively. The distribution of maximum wave height is multipeaked. Here also the distribution is broad ranged. The modified Weibull curve is also normal and it simulates the distribution of maximum wave heights of all range.

September

The maximum wave height ranges from 0.3m to 2.1m. The scale parameter 'a' and the shape parameter 'b' have the values 1.046 and 2.097 respectively. The distribution of maximum wave height is multipeaked. A single peak in the higher wave range may be due to the creative interference of sea waves from different generating areas. Here also the modified Weibull curve is normal.

October

The maximum wave height ranges from 0.15m to 1.05m. The scale parameter 'a' and the shape parameter 'b' have the values 0.545 and 2.879 respectively. The distribution of maximum wave height is multipeaked. Here also the modified Weibull curve is normal and it simulates the maximum wave height distribution of all range.

3.2 Prediction of conventional significant wave height

Distribution of digitised zero up-crossing wave heights from wave recorder charts by zero up-crossing technique for the month of January'1981 off valiathura are simulated by the depth factor accommodated Weibull model. The result is far better than other data because of its accuracy. By using this data significant wave heights are predicted using the equation (2.7a). The results are given in Table D. It is found that the predicted conventional significant wave height is comparable with the observed value, in some cases even for two decimal places. A maximum deviation of only 0.21m and a minimum deviation of 0.00m are obtained. A shoaling coefficient is also included in the prediction formulae for estimating the significant wave height, which give more accurate results. This stems from the fact that since conventional significant wave height is the average of the one-third highest waves, the higher side of the wave height distribution is influenced by both depth factor and shoaling coefficient.

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